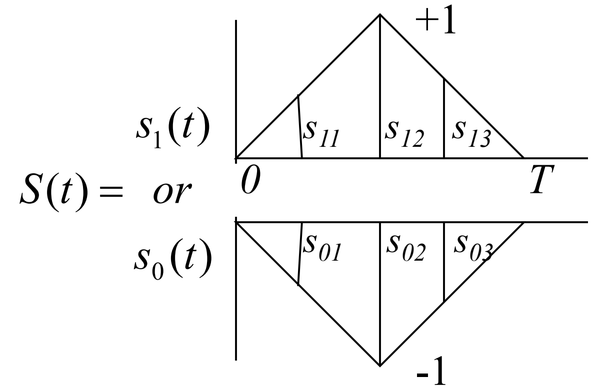
Binary Detection: Multiple Independent Observations

- Consider the example in which we have triangular pulses and we use 3 samples as shown in Figure
- The decision variable is



$$\bar{S}_1 = \begin{bmatrix} 0.5 \\ 1.0 \\ 0.5 \end{bmatrix}; \bar{S}_0 = \begin{bmatrix} -0.5 \\ -1.0 \\ -0.5 \end{bmatrix}; \bar{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix}$$

$$Z = \bar{Y}^T \bar{S}_1 - \bar{S}_0 = Y_1 + 2Y_2 + Y_3 :$$

$$Z\,|'1' = (0.5 + N_1) + 2(1 + N_2) + (0.5 + N_3) = 3 + N_1 + 2N_2 + N_3 \qquad Y_i \,|\, H_0 = s_{0i} + N_i$$

$$Z\,|'0' = (-0.5 + N_1) + 2(-1 + N_2) + (-0.5 + N_3) = -3 + N_1 + 2N_2 + N_3 \qquad Y_i \,|\, H_1 = s_{1i} + N_i$$

- The mean and variance of the decision variables are:

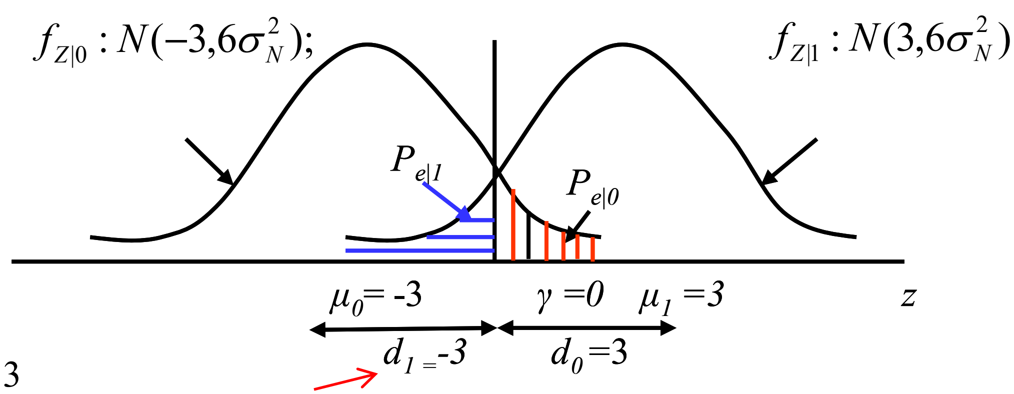
$$\mu_1 = 3; \quad \sigma_Z^2 = 6\sigma_N^2; \qquad \mu_0 = -3; \quad \sigma_Z^2 = 6\sigma_N^2$$

$$f_{Z|1} : N(3, 6\sigma_N^2); f_{Z|0} : N(-3, 6\sigma_N^2);$$

- The decision threshold is $\gamma = 0$
- The probability of error is shown in the figure as

$$P_e = P_0 P_{e1} + P_0 P_{e0}$$

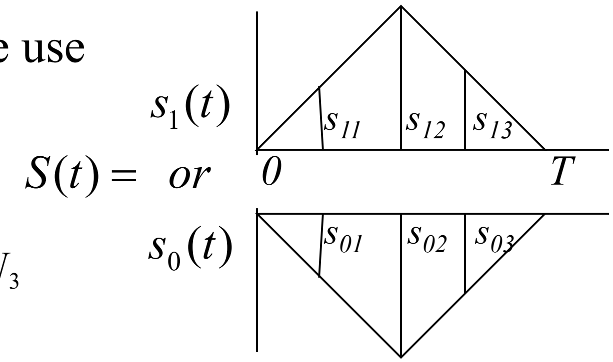
$$= Q(3/\sqrt{6\sigma_N^2}) = Q(\sqrt{1.5/\sigma_N^2})$$



Binary Detection: Multiple Independent Observations : Sub-Optimal Decision rule

- Instead of the optimal decision variable, suppose we use

$$Z = Y_1 + Y_2 + Y_3 :$$
$$Z\,|'1' = (0.5 + N_1) + (1 + N_2) + (0.5 + N_3) = 2 + N_1 + N_2 + N_3$$
$$Z\,|'0' = (-0.5 + N_1) + (-1 + N_2) + (-0.5 + N_3) = -2 + N_1 + N_2 + N_3$$



$$\bar{S}_1 = \begin{bmatrix} 0.5 \\ 1.0 \\ 0.5 \end{bmatrix}; \bar{S}_0 = \begin{bmatrix} -0.5 \\ -1.0 \\ -0.5 \end{bmatrix}; \bar{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix}$$

- The mean and variance of the decision variables are:

$$\mu_1 = 2;\ \ \sigma_Z^2 = 3\sigma_N^2; \qquad \mu_0 = -2;\ \ \sigma_Z^2 = 3\sigma_N^2$$
$$f_{Z|1} : N(2, 3\sigma_N^2); f_{Z|0} : N(-2, 3\sigma_N^2);$$

- The decision threshold is $\gamma = 0$
- The probability of error is shown in the figure as

$$P_e = P_0 P_{e1} + P_0 P_{e0}$$
$$= Q(2/\sqrt{3\sigma_N^2}) = Q(\sqrt{1.33/\sigma_N^2})$$
$$> Q(\sqrt{1.5/\sigma_N^2})$$

[The optimum MAP decision rule is better as expected]